Simple 3d SCFTs and simple 3-manifolds

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Based on
ArXiv: 1610.09259 with Jin-beom Bae (KIAS) and Jaehoon Lee (UBC)
(+ work in progress with Kazuya Yonekura and Yuji Tachikawa (Kavli-IPMU))

Part I : Simple 3d SCFTs

Simple 3d SCFTs

- $3d\ \mathcal{N}=2$ Superconformal field theories
  - Algebra : $osp(2|4) \supset SO(2,3)\times U(1) \supset SO(2)\times SO(3)\times U(1)$
  - Fermionic generators : $Q, \bar{Q}, S, \bar{S}$. $\quad \Delta \quad j \quad r$
  - Local operator will be labeled by $[j]_{\Delta}^{(r)} \in \mathcal{H}_{S^2}$
  - Basic $3d\ \mathcal{N}=2$ SUSY multiplets : chiral multiplet : $\Phi\ (\phi, \psi, F)$, vector multiplet : $V\ (A_\mu, \lambda, \sigma, D)$

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- "Simple" $3\text{d}\ \mathcal{N}=2$ SCFTs
  - Technically, *simple* means

    or $\begin{cases} \text{small } F\ (:= -\log|Z_{S^3}|),\ F-\text{theorem } (F_{UV} \geq F_{IR}) \\ \text{small } C_T\ (\langle T(x)T(0)\rangle \propto C_T |x|^{-6}),\ \text{conformal bootstrap} \end{cases}$

  - In this talk,

    simple SCFTs := SCFTs obtained by "deforming" $\mathcal{T}_{\mathbb{I}}$ ("neighborhood of free chiral")

    $\mathcal{T}_{\mathbb{I}} := \{$ a theory of single free chiral $\}$

  - Motivation : "simpler" $\simeq$ "more universal"

    ex) *cWZ* model ($W \sim \Phi^3$) in condensed matter [Grover,Sheng,Vishwanath: '13]

Deformations I

- ($SUSY$ preserving) deformations of a $3d$ $\mathcal{N}=2$ $SCFT$ $\mathcal{T}$
  - Superpotential — *When chiral multiplet* $\subset \mathcal{H}_{S^2}(\mathcal{T})$
  - Gauging (+Chern-Simons term)
  - Real mass (or FI-term) deformation

  } *When conserved current multiplet* $\subset \mathcal{H}_{S^2}(\mathcal{T})$

Chrial multiplet : $[0]^{(r)}_{\Delta=r} \oplus [1/2]^{(r-1)}_{\Delta=r+1/2} \oplus [0]^{(r-2)}_{\Delta=r+1}$ and its conformal descendents $(\partial O, \partial^2 O, \ldots)$

$$[0]^{(r)}_{\Delta=r} \xrightarrow{Q} [1/2]^{(r-1)}_{\Delta=r+1/2} \xrightarrow{Q} [0]^{(r-2)}_{\Delta=r+1}$$

$ex)\ \mathcal{T}_{\mathbb{I}}$ $\quad \phi^{n\geq 2}\ ([0]^{(n/2)}_{\Delta=n/2})$

Conserved current multiplet : $[0]^{(0)}_{\Delta=1}$ $\quad [1]^{(0)}_{\Delta=2} \oplus [0]^{(0)}_{\Delta=2}$ with conformal descendents

$$[0]^{(0)}_{\Delta=1} \xrightarrow{\overline{Q}} [1/2]^{(1)}_{\Delta=3/2} \xrightarrow{Q} [1]^{(0)}_{\Delta=2} \oplus [0]^{(0)}_{\Delta=2}$$

$$[0]^{(0)}_{\Delta=1} \xrightarrow{Q} [1/2]^{(-1)}_{\Delta=3/2} \xrightarrow{\overline{Q}} [1]^{(0)}_{\Delta=2} \oplus [0]^{(0)}_{\Delta=2}$$

$ex)\ \mathcal{T}_{\mathbb{I}}$ $\quad \phi\phi^{*}([0]^{(0)}_{\Delta=1})$

Deformations I

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  } *When conserved current multiplet* $\subset \mathcal{H}_{S^2}(\mathcal{T})$

- Superpotential deformation on $\mathcal{T}_{\mathbb{I}}$

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\delta W = \Phi^{n\geq 2}} \begin{cases} \varnothing\,, & n=2 \text{ (mass)} \\ \mathcal{T}_{cWZ} & n=3 \text{ (relevant)} \\ \mathcal{T}_{\mathbb{I}} & n\geq 4 \text{ (not exactly marginal } or \text{ irrelevant)} \end{cases}$$

- Real mass deformation on $\mathcal{T}_{\mathbb{I}}$

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\text{real mass}} \varnothing \ or \text{ topological theory (abelian pure CS theory)}$$

Deformations II

- Witten's $SL(2,\mathbb{Z})$: systematic way of gauging $U(1)$ [Witten :`03]

$\mathcal{T}$:= a 3d SCFT with a $U(1)$ flavor symmetry

$$\mathcal{T} \xrightarrow{S=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} S\cdot\mathcal{T}_\varphi := \text{Gauging } U(1) \text{ of } \mathcal{T}_\varphi \ \ (\text{Toplogical } U(1)_J : \ J := *F \ \Rightarrow \partial_\mu J^\mu = 0)$$

$$\mathcal{T} \xrightarrow{T=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}} T\cdot\mathcal{T} := \text{adding background CS-term } +1 \, .$$

$$SL(2,\mathbb{Z}) : S^2 = C \text{ (charge conj) } , S^4 = \mathbb{I} \, , \quad (ST)^3 = \mathbb{I} \, .$$

- $\mathcal{T}_{(p,q)}$

$\mathcal{T}_{\mathbb{I}}$:= a free chrial theory with background CS level $-1/2$, $\mathcal{T}_\varphi := \varphi\cdot\mathcal{T}_{\mathbb{I}}$

$$\mathcal{T}_{(p,q)} := \mathcal{T}_\varphi \ \text{ with } \ \varphi = \begin{pmatrix} p & q \\ * & * \end{pmatrix}$$

Dualities among $\mathcal{T}_{(p,q)}$

- Basic Mirror pair

$$\Phi = (U(1)_{1/2} + \Phi\ )$$

V_+ become gauge-invariant
$\Delta(V_+) = 1/2$ (localization, F − *maximization*)
$\Rightarrow$ (*a* free chiral theory)

- Dualities among $\mathcal{T}_{(p,q)}s$

$$\mathcal{T}_{ST} = \mathcal{T}_{\mathbb{I}}$$

$$\Rightarrow \ \mathcal{T}_{\varphi \cdot ST} = \mathcal{T}_{\varphi \cdot \mathbb{I}} \text{ for all } \varphi \in SL(2,\mathbb{Z})$$

$$\Rightarrow \ \mathcal{T}_{\begin{pmatrix} p & q \\ * & * \end{pmatrix} \cdot \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}} = \mathcal{T}_{\begin{pmatrix} p & q \\ * & * \end{pmatrix}}$$

$$\Rightarrow \mathcal{T}_{p,q} = \mathcal{T}_{q-p,-p} \qquad C \cdot \mathcal{T}_{p,q} = \mathcal{T}_{-p,-q} \qquad P \cdot \mathcal{T}_{p,q} = \mathcal{T}_{q,p}$$

SUSY partition functions

- Partition functionon ellipsoid S_b^3 [Hama,Hosomichi,Lee :`11]

$$Z_b[\mathcal{T}_{(p,q)};X]=\frac{1}{\sqrt{q}}\int\frac{dY}{2\pi b}e^{\frac{sX^2}{4\pi ib^2q}-\frac{XY}{2\pi ib^2q}+\frac{pY^2}{4\pi ib^2q}}\psi_b(Y)\ ,\quad (ps-1\in q\mathbb{Z}\ ,\psi_b:\text{quantum dilogarithm})$$

$$X=2\pi bm+2\pi i(1+b^2)R_{(p,q)}\ ,\quad R_{(p,q)}:=p\Delta_\Phi+q\frac{1-\Delta_\Phi}{2}$$

Conformal R-charge $R^*_{(p,q)}$ is determined by F-maximization

$$\left|Z_{b=1}[\mathcal{T}_{(p,q)};X=4\pi iR^*_{(p,q)}]\right|\leq\left|Z_{b=1}[\mathcal{T}_{(p,q)};X=4\pi iR_{(p,q)}]\right|$$

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Conformal R-charge $R^*_{(p,q)}$ is determined by F-maximization

- Superconformal index [Kim :`09]

$$\mathcal{I}_{(p,q)}(u;x)=\sum_{e\in\mathbb{Z}}\mathcal{I}_\Delta(-qe,pe;x)((-x^{1/2})^{R^*_{(p,q)}}u)^e\ .\qquad \left(\mathcal{I}_\Delta(m,e;x):=\sum_{n=[e]}^{\infty}\frac{(-1)^n x^{\frac{1}{2}n(n+1)-(n+\frac{1}{2}e)m}}{(x)_n(x)_{n+e}}\right)$$

- Non-trivial test of duality

$$Z_b[\mathcal{T}_{p,q};X=2\pi i(1+b^2)R^*_{(p,q)}]=Z_b[\mathcal{T}_{q-p,-p};X=2\pi i(1+b^2)R^*_{(q-p,-p)}]\ ,\ldots$$

Further deformation on $\mathcal{T}_{(p,q)}$

- No relevant chiral multiplet in $\mathcal{T}_{(\mathrm{p,q})}$
  - except $(p,q)=(1,0)$,$(-1,0)$,$(0,-1)$, $(-1,-1)$, $(1,1)$: $\mathcal{T}_{\mathbb{I}}$, $C\mathcal{T}_{\mathbb{I}}$, $P\mathcal{T}_{\mathbb{I}}$
  - Superconformal Index , semi-classical anlysis

    $(a_{\Phi}^{\dagger})^{k}\left|\textit{monopole of}\ \text{charge}\ n\right\rangle \in \mathcal{H}_{S^2}(\mathcal{T}_{p,q})$

    carry non-zero spin *unless* $n=0$ or $k=0$

    Unless *quantum* miracle, *no relevant chiral primary*

    $[\text{long}]_{\text{semi-classical}} \to [\text{short}]_1 \oplus [\text{short}]_2 \oplus .....$

    (marginal *or* irrelevant) [Cordova, intrilligator :`16]
  - *So*, no superpotential deformation

- Real mass deformation to topological theory

Summary of Part I

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\delta W = \Phi^3} \mathcal{T}_{cWZ}$$ **(no flavor symmetry)**

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\varphi = \begin{pmatrix} p & q \\ * & * \end{pmatrix}} \mathcal{T}_{(p,q)}$$ **(U(1) symmetry)**

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\text{other deformation}} \varnothing$$ or topological theory

Part II : Simple 3-manifolds

Curious observation

- Taking $b \to 0$ limit ($\hbar \coloneqq 2\pi i b^2$)

$$Z_b[\mathcal{T}_{(p,q)}; X=0] = \frac{1}{\sqrt{q}} \int \frac{dY}{2\pi b} \exp\left[\frac{pY^2}{4\pi i b^2 q}\right] \psi_b(Y)$$

$$\xrightarrow{b\to 0} \frac{1}{\sqrt{q}} \int \frac{dY}{2\pi b} \exp\left[\frac{1}{\hbar}\left(\frac{pY^2}{2q} + \mathrm{Li}_2(e^{-Y})\right) + \dots\right]$$

$$\xrightarrow{\text{saddle-point approx}} \exp\left[\frac{1}{\hbar} S^{(0)}(p,q) + \dots\right]$$

$$\mathrm{Im}[S^{(0)}(-2,1)] = -\underline{0.942707}$$

$$\mathrm{Im}[S^{(0)}(4,1)] = -\underline{0.981369} \quad (X = 2\pi i(1+b^2))$$

$$\mathrm{Im}[S^{(0)}(-1,1)] = -\underline{1.01494}$$

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closed hyperbolic 3-manifolds	hyperbolic volume	length spectrum
1 	vol = 0.94270736 CS = 0.06004307 $H_1 = \mathbb{Z}_5 + \mathbb{Z}_5$ Isom = D_6	(×3) $0.58460369 + 2.49537046\,i$ * (×3) $0.79413466 - 2.30485682\,i$
2 	vol = 0.98136883 CS = 0.07703818 $H_1 = \mathbb{Z}_5$ Isom = D_2	(×1) $0.57808244 + 2.13243064\,i$ (×1) $0.72156837 - 1.15121299\,i$ * (×2) $0.88944300 + 2.94185905\,i$ (×2) $0.99832519 - 2.92101779\,i$
3 	vol = 1.01494161 CS = 0 $H_1 = \mathbb{Z}_3 + \mathbb{Z}_6$ Isom = S_{16}	(×2) $0.83144295 - 1.94553076\,i$ (×2) $0.83144295 + 1.94553076\,i$ (×2) $0.86255463 - 2.68067319\,i$ * (×2) $0.86255463 + 2.68067319\,i$ *

from [Craig D. Hodgson and Jeffrey R. Weeks,'94]

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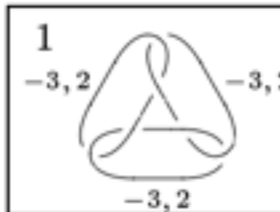
$$\langle W_Q(p,q;b)\rangle \xrightarrow{b\to 0} \exp\left[\frac{\ell_Q(p,q)}{2}\right] + \exp\left[-\frac{\ell_Q(p,q)}{2}\right]$$

$$\ell_{Q=-1}(-2,1) = 0.584604 - 2.49537\, \mathrm{i}$$

$$\ell_{Q=1}(-2,1) = 0.794135 + 2.30486\, \mathrm{i}$$

closed hyperbolic 3-manifolds	hyperbolic volume	length spectrum
1 	vol = 0.94270736 CS = 0.06004307 $H_1 = \mathbb{Z}_5 + \mathbb{Z}_5$ Isom = D_6	(×3) 0.58460369 + 2.49537046 i ∗ (×3) 0.79413466 − 2.30485682 i
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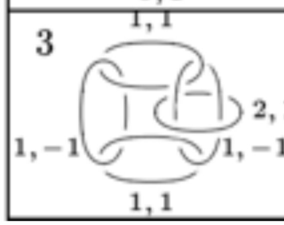
3 	vol = 1.01494161 CS = 0 $H_1 = \mathbb{Z}_3 + \mathbb{Z}_6$ Isom = S_{16}	(×2) 0.83144295 − 1.94553076 i (×2) 0.83144295 + 1.94553076 i (×2) 0.86255463 − 2.68067319 i ∗ (×2) 0.86255463 + 2.68067319 i ∗

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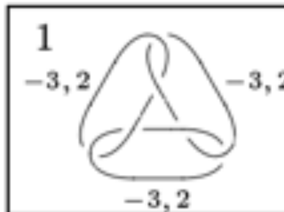
1 	vol = 0.94270736 CS = 0.06004307 $H_1 = \mathbb{Z}_5 + \mathbb{Z}_5$ Isom = D_6	$(\times 3)$ 0.58460369 + 2.49537046 i * $(\times 3)$ 0.79413466 − 2.30485682 i
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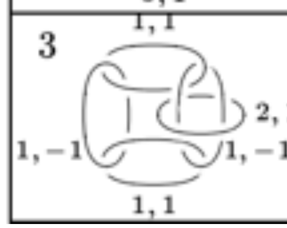
3 	vol = 1.01494161 CS = 0 $H_1 = \mathbb{Z}_3 + \mathbb{Z}_6$ Isom = S_{16}	$(\times 2)$ 0.83144295 − 1.94553076 i $(\times 2)$ 0.83144295 + 1.94553076 i $(\times 2)$ 0.86255463 − 2.68067319 i * $(\times 2)$ 0.86255463 + 2.68067319 i *

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$$\ell_{Q=-1}(-2,1) = 0.584604 - 2.49537\, i$$

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$$S^{(0)}(-2,1) = -1.1852 - 0.942707 i = S_{CS}^{(0),\overline{\text{hyp}}}(M = \text{Weeks}),$$

$$S^{(1)}(-2,1) = -0.767859 + 0.923278 i = S_{CS}^{(1),\overline{\text{hyp}}}(M = \text{Weeks}),$$

$$S^{(2)}(-2,1) = 0.0462791 + 0.0684314 i = S_{CS}^{(2),\overline{\text{hyp}}}(M = \text{Weeks}),$$

$$S^{(3)}(-2,1) = -0.036626 + 0.0111375 i = S_{CS}^{(3),\overline{\text{hyp}}}(M = \text{Weeks}).$$

$$\int [d\mathcal{A}] \exp\left(\frac{i}{2\hbar}\int_M \mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right) \xrightarrow{\hbar\to 0,\ \mathcal{A}=\mathcal{A}^{\overline{hyp}}+\delta\mathcal{A}}$$

$$\exp\left(\frac{1}{\hbar} S_{CS}^{(0),\overline{\text{hyp}}}(M) + S_{CS}^{(1),\overline{\text{hyp}}}(M) + \hbar S_{CS}^{(2),\overline{\text{hyp}}}(M) + \dots\right)$$

$$\mathcal{A}^{\overline{hyp}} := \omega_{\text{hyp}} - i\, e_{\text{hyp}}$$

[Bae,DG,Lee :`16]

Curious observation

$$\mathcal{T}_{-2,1} \coloneqq (U(1)_{-5/2} + \Phi)$$

$$\mathcal{T}_{4,1} \coloneqq (U(1)_{7/2} + \Phi)$$

$$\mathcal{T}_{-1,1} \coloneqq (U(1)_{-3/2} + \Phi)$$

How??

- There exists *a 6d theory* $\mathcal{T}_{6d}$ s.that

$$\mathcal{T}_{6d} \xrightarrow{\text{a compactification on } M = (\text{Weeks})} \mathcal{T}_{(-2,1)}$$

- $\mathcal{T}_{6d}$ has at least $SO(3)_R$ global symmetry (for top-twisting along M)
- $\mathcal{T}_{6d}$ *has* maximally *SUSY* (16*Q*)

  $Q \in \mathbf{2}$ of $\mathrm{SO}(3)_{\mathrm{M}}$, so $Q \in \mathbf{2}$ of $SO(3)_R$ which gives $\frac{1}{4}$ *BPS* top-twisting
- $\mathcal{T}_{6d}$ *is* A_1 (2,0) theory!

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  - $\mathcal{T}_{6d}$ *is* A_1 $(2,0)$ theory!

- 3d/3d correspondence

$$(A_1 \ (2,0) \text{ theory}) \xrightarrow{\text{a compactification on } M} T[M]$$

$$\{\text{BPS ptns of } T[M]\} = \{SL(2,\mathbb{C}) \text{ complex CS ptns on } M\}$$

[Lee,Yamazaki: `13]
[Cordova, Jafferis: `13]

from " $6d\ (2,0)$ *on* $S^1 \to 5d\ SYM$ "

Loop operators $\longleftrightarrow$ Wilson loops

from " codimension 4 *in* $6d$ $\to$ Wilson loops *in* $5d$ "

[DG, Romo, Kim, Masahito: `15]

3d/3d correspondence for CH3

- 3d/3d correspondence

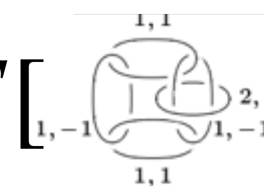
$$(A_1\ (2,0)\ \text{theory}) \xrightarrow{\text{a compactification on } M} T[M]$$

$$\{\text{BPS ptns of } T[M]\} = \{SL(2,\mathbb{C}) \text{ complex CS ptns on } M\}$$

- first examples of $T[M]$, M = closed hyperbolic 3-manifolds (CH3)

$T[$ 

$] = \mathcal{T}_{-2,1}$, $T[$ 

$] = \mathcal{T}_{4,1}$, $T[$ 

$] = \mathcal{T}_{-1,1}$

- Symmetry enhancement

$$6d\ SO(5)_R \to SO(3)_R \times SO(2)_R \xrightarrow{top \text{ twisting}} SO(2)_R \text{ (no flavor sym)}$$

But $\mathcal{T}_{p,q}$ *has* $U(1)$ *sym*! $cf)$ 2 $M5s$ *on* genus two Riemman surface

$$\{\text{BPS ptns of } T[M]\} = \{\text{Refined(?) } SL(2,\mathbb{C}) \text{ complex CS ptns on } M\}$$

Summary & Discussion

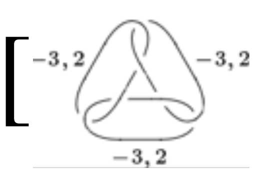
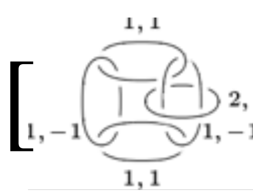
Neighborhood SCFTs of free chiral theory

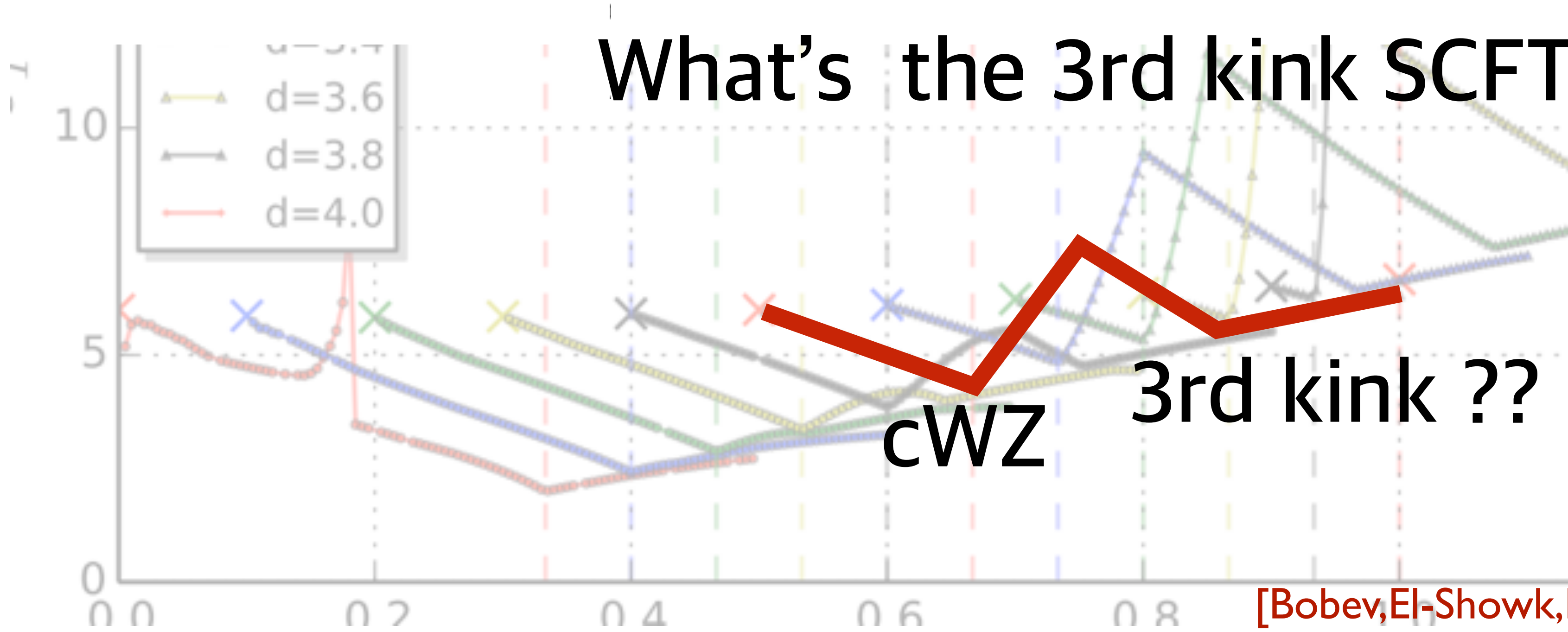
$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\delta W = \Phi^3} \mathcal{T}_{cWZ}$$ **(no flavor symmetry)**

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\varphi = \begin{pmatrix} p & q \\ * & * \end{pmatrix}} \mathcal{T}_{(p,q)}$$ **(U(1) symmetry)**

$$\mathcal{T}_{\mathbb{I}} \xrightarrow{\text{other deformation}} \mathcal{T}_{\mathbb{I}}$$ or topological theory

Wrapped two M5s on small CH3 (U(1) symmetry enhancement)

$T[$  $] = \mathcal{T}_{-2,1}$, $T[$  $] = \mathcal{T}_{4,1}$, $T[$  $] = \mathcal{T}_{-1,1}$



[Bobev,El-Showk,Mazac,Paulos: '15]